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A relativistic study of the nucleon helicity amplitudes*

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Abstract. We perform a calculation of the relativistic transition form factors for the electromagnetic excitation of the nucleon resonances. We use as input the 3-quark wave functions obtained in a Constituent Quark Model with three-body forces in the hypercentral approach. With respect to the non relativistic calculations a significant contribution is obtained up to $Q^2 \simeq 2(GeV/c)^2$. However, the low Q^2-behaviour exhibits a lack of strength, which may be connected with the need of taking into account explicitly further degrees of freedom beyond the three constituent quark ones.

PACS. 12.39.Ki Relativistic quark model – 13.40.Gp Electromagnetic form factors – 14.20.Gk Baryon resonances with $S = 0$

1 Introduction

The non relativistic constituent quark models (CQM) have given good results in the study of the static properties of the nucleon [1,2], but they are unable to reproduce the Q^2 behaviour of the electromagnetic form factors even in the low momentum transfer [3–7]. The inclusion of relativistic effects is therefore expected to be important.

The structure of the electromagnetic current of a relativistic bound system is still an unsolved problem. There have been recently several calculations of relativistic corrections to the electromagnetic form factors of the nucleon within constituent quark models. Three main lines have been followed: the expansion of relativistic current operators in powers of the inverse quark mass, $\frac{1}{m}$ [8–11], the evaluation of the current matrix elements in a light-cone approach [5,12,13] and the expansion of the full relativistic current matrix elements in powers of $\frac{1}{m}$ [14–16].

In this work we follow the last method which has been already used in [16] for the elastic nucleon form factors. We generalize to the transition electromagnetic form factors of the nucleon the simplified approach of [16] which is useful for a preliminary calculation of the relativistic corrections and we apply it for the excitation of the negative parity resonances. The use of Lorentz boosts for the quark spinors ensures that the relation between the dynamic variables of the initial and final states is relativistically correct. On the other hand, we assume that quark internal motion is well described by standard non relativistic wave functions. For the calculations, we use as input the 3q-wave functions of [17]. The current matrix elements are constructed with a quark current operator containing only one-body terms and no quark form factors are introduced. We point out that the non relativistic expansion of the matrix elements of the present work, up to order m^{-2}, is coincident with that given by standard procedures [18, 19,15] introduced for few-nucleon systems and no approximation is done with respect to the momentum transfer Q^2 dependence.

In Sec. 2, we describe the non relativistic constituent quark model of [17]; in Sec. 3 we present the evaluation of the current matrix elements arriving at simple analytical expressions for the form factors. In Sec. 4, we discuss the results and make a comparison with the experimental data. A brief conclusion is given in Sec. 5.

2 The model

We give a brief account of the non-relativistic constituent quark model in the hypercentral approach proposed in [17]. After having removed the center of mass coordinate, the internal quark motion is described by the Jacobi coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$:

$$\begin{aligned} \boldsymbol{\rho} &= \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2), \\ \boldsymbol{\lambda} &= \frac{1}{\sqrt{6}}(\boldsymbol{r}_1 + \boldsymbol{r}_2 - 2\boldsymbol{r}_3) \end{aligned} \tag{1}$$

or equivalently, ρ, Ω_ρ, λ, Ω_λ. The three-quark dynamics is conveniently described in the hyperspherical harmonic

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formalism [20–22]. To this end one introduces the hyperspherical coordinates, which are obtained substituting the absolute values ρ and λ by

$$x = \sqrt{\rho^2 + \lambda^2}, \qquad \xi = arctg(\frac{\rho}{\lambda}), \tag{2}$$

where x is the hyperradius and ξ the hyperangle. The quark potential, V, is assumed to be hypercentral, that is to depend on the hyperradius x only. Therefore, $V = V(x)$ is in general a three-body potential, since the hyperradius x depends on the coordinates of all the three quarks. However, $V(x)$ contains also contributions from two-body potentials in hypercentral approximation [22,23]. The Schrödinger equation in the hyperspherical coordinates is, for hypercentral potentials, simply reduced to a single equation for the hyperradial part of the 3q-wave function, since the angular and hyperangular parts of the 3q-states are factored out and are given by the known hyperspherical harmonics [20]. The h.o. potential, which is widely used in quark models because of its analytical solution, can be treated also in the hyperspherical formalism. In fact, it turns out to be exactly hypercentral, since

$$\sum_{i<j} \frac{1}{2} k(\boldsymbol{r_i} - \boldsymbol{r_j})^2 = \frac{3}{2} k x^2 = V_{h.o}(x) \tag{3}$$

There is at least another potential which leads to analytical three-quark wave functions, that is the 'hypercoulomb' potential [24,25,22,26]

$$V_{hyc}(x) = -\frac{\tau}{x}. \tag{4}$$

This potential is not confining, however it has interesting properties. It leads to a power-law behaviour of the proton form factor [24,25] and of all the transition form factors [6] and has a perfect degeneracy between the first 0^+ excitated state and the first 1^- states [27,24,28,26], which can be respectively identified with the Roper resonance and the negative parity resonances. This degeneracy seems to be in agreement with phenomenology and is typical of an underlying O(7) symmetry [26,6].

According to the analysis of [17,29,7], one can give a good description of the non-strange baryon spectrum, the photocouplings and the electromagnetic transition form factors using a three-body potential of the form

$$V(x) = -\frac{\tau}{x} + \beta x. \tag{5}$$

The hypercentral equation is solved numerically and, starting from the corresponding hyperradial wave functions, one can construct a complete basis of antisymmetric three-quark states, analogously to what is done in standard h.o. models [1], combining the $SU(6)$-spin-flavour configurations with the space wave functions [17]. In order to account for the splitting within each $SU(6)$-multiplet, one can introduce a hyperfine interaction, which is treated as a perturbation and therefore each resonance is a superposition of $SU(6)$-configurations. In this way, one obtains [17] model wave functions for the three-quark states which are fixed by fitting the energy spectrum and which can then be used for parameter free calculations of any further baryon property, as for instance the electromagnetic transiton form factors reported below.

3 Electromagnetic transition form factors

The electromagnetic transition form factors, $A_{1/2}$ and $A_{3/2}$, are defined as the transition matrix element of the transverse electromagnetic interaction between the nucleon, N, and the resonance, R, states:

$$\begin{aligned} A_{1/2} &= \sqrt{\tfrac{2\pi\alpha}{k_W}} \langle \Psi_R, J', J'_z = \tfrac{1}{2} \,|\epsilon^+_\mu \textstyle\sum_{i=1}^3 j^\mu(i) \\ &\qquad \Psi_N, J = \tfrac{1}{2}, J_z = -\tfrac{1}{2}\rangle \\ A_{3/2} &= \sqrt{\tfrac{2\pi\alpha}{k_W}} \langle \Psi_R, J', J'_z = \tfrac{3}{2} \,|\epsilon^+_\mu \textstyle\sum_{i=1}^3 j^\mu(i) \\ &\qquad \Psi_N, J = \tfrac{1}{2}, J_z = \tfrac{1}{2}, \rangle \end{aligned} \tag{6}$$

where $j^\mu(i)$ is the e.m. current of the i-th quark and the photon polarization vector is $\epsilon^+_\mu = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$; α is the fine structure constant and

$$k_W = \frac{M_R^2 - M_N^2}{2M_R}. \tag{7}$$

We shall perform the calculations in the so called Equal Velocity Frame (EVF) which is defined by the following condition

$$\frac{\boldsymbol{P}_R}{M_R} = -\frac{\boldsymbol{P}_N}{M_N}, \tag{8}$$

where $\boldsymbol{P}_N$ and $\boldsymbol{P}_R$ are the three-momenta of the nucleon and of the resonance respectively, and M_N and M_R are their masses. In the elastic case, the EVF coincides with the Breit frame.

In order to calculate the current matrix elements of (6) we introduce the quark-momenta, $\boldsymbol{p}_i$, in a generic reference frame, which are related to the intrinsic ones, $\boldsymbol{p^*}_i$ (that is in the baryon rest frame), by the Lorentz transformations

$$\boldsymbol{p}_i = \boldsymbol{p^*}_i + \frac{\boldsymbol{P}}{M}\Big[\frac{\boldsymbol{P}}{M+E}\boldsymbol{p^*}_i + \epsilon^*_i\Big], \tag{9}$$

where ϵ^*_i is the quark energy in the baryon rest frame; $M = \sum_{i=1}^3 \epsilon^*_i$ is the baryon mass, E and $\boldsymbol{P}$ are the baryon total energy and momentum in the new reference frame, respectively. We assume that the virtual photon is absorbed by a single quark, that for symmetry reasons can be taken as the third one. We apply the Lorentz transformation of (9) to the intrinsic quark momenta in the nucleon and in the resonance states separately. Then we use the three-momentum conservation and assume $\epsilon^*_i \simeq \frac{M_N}{3}$ for the nucleon and $\epsilon^*_i \simeq \frac{M_R}{3}$ for the resonance and, considering that the boost is parallel to the photon momentum $\boldsymbol{q}$, we have

$$-\sqrt{\frac{2}{3}}\frac{E_R}{M_R}p'_{\lambda_q} + \frac{P_R}{3} = -\sqrt{\frac{2}{3}}\frac{E_N}{M_N}p_{\lambda_q} + \frac{P_N}{3} + q, \tag{10}$$

$$\frac{E_R}{M_R} p'_{\rho_q} = \frac{E_N}{M_N} p_{\rho_q}, \quad (11)$$

$$\boldsymbol{p'}_{\lambda_\perp} = \boldsymbol{p}_{\lambda_\perp}, \quad (12)$$

$$\boldsymbol{p'}_{\rho_\perp} = \boldsymbol{p}_{\rho_\perp}, \quad (13)$$

where the indices q and $\perp$ mean the parallel and the orthogonal components to $\boldsymbol{q}$, respectively, the apex is used to denote the intrinsic momenta of the resonance state and $\boldsymbol{p}_\rho$ and $\boldsymbol{p}_\lambda$ are conjugate to the Jacobi coordinates of (1).

Since in the EVF

$$\frac{E_N}{M_N} = \frac{E_R}{M_R}, \quad (14)$$

we have

$$p'_{\lambda_q} = p_{\lambda_q} - \sqrt{\frac{2}{3}} q_{eff}, \quad (15)$$

$$\boldsymbol{p'}_{\lambda_\perp} = \boldsymbol{p}_{\lambda_\perp}, \quad (16)$$

$$\boldsymbol{p'_\rho} = \boldsymbol{p_\rho}, \quad (17)$$

where q_{eff} is defined as

$$q_{eff} = \frac{q}{\gamma}, \quad (18)$$

and γ is the ratio of Eq. 14

$$\gamma = \frac{E_N}{M_N} = \frac{E_R}{M_R}. \quad (19)$$

We note that in the EVF the transformation eqs. (15), (16) and (17) formally coincide with the ones obtained in the elastic case [16] and so it is a convenient choice for the calculations.

The 3-quark state wave function is obtained by applying the standard Dirac boost operator that transforms the quark spinors $u_i(p_i^*)$ from the nucleon rest frame to the EVF

$$\Psi_N = \prod_{i=1}^{3} B_i u_i(p_i^*) \phi(\boldsymbol{p_\rho}, \boldsymbol{p_\lambda}). \quad (20)$$

and similarly for the resonance, Ψ_R; B_i is the Dirac boost operator corresponding to the velocities of the nucleon and of the resonance, which in both cases have the absolute value

$$v = \frac{\sqrt{\gamma^2 - 1}}{\gamma}. \quad (21)$$

In (20), $\phi(\boldsymbol{p_\rho}, \boldsymbol{p_\lambda})$ is the standard non relativistic 3q-wave function, where for simplicity of notation, we have omitted the spin and isospin variables. The quark boosted spinors $\psi_i = B_i u_i(p_i^*)$ are normalized with the invariant condition

$$\overline{\psi}_i \psi_i = 1. \quad (22)$$

The current operator of the i-th quark, $j^\mu(i)$, has the form

$$j^\mu(i) = \sqrt{\frac{m}{\epsilon'_i}} \gamma_i^\mu \sqrt{\frac{m}{\epsilon_i}}, \quad (23)$$

where m is the quark mass and ϵ_i (ϵ'_i) is the initial (final) quark energy in the EVF. The normalization factors $\sqrt{\frac{m}{\epsilon'_i}}$, $\sqrt{\frac{m}{\epsilon_i}}$, have been introduced in order to obtain for the current matrix elements the correct expansion in powers of $\frac{1}{m}$ (i.e. coincident with what is usually quoted in the literature) as shown in [19].

The resulting expression for the current matrix element is complicated because of the presence of non-local terms coming from the momentum dependence and the calculations can be performed numerically. However, in order to arrive at a preliminary calculation of the relativistic effects for the e.m. currents we introduce some simplifying assumptions.

Consistently with the use of a non relativistic model for the internal nucleon dynamics, we approximate the quark energies in the baryon rest frame as $\epsilon_i^* \simeq m$, where m is the quark constituent mass. We perform an expansion keeping contributions up to first order in the relative quark momenta, but we treat exactly the dependence on the momentum transfer $\boldsymbol{q}$. To this end, we introduce in the matrix element of (6) the variable π_λ that is related to p_{λ_q} and p'_{λ_q} in the following way

$$p'_{\lambda_q} = \pi_\lambda - \frac{1}{2}\sqrt{\frac{2}{3}} q_{eff} \quad (24)$$

$$p_{\lambda_q} = \pi_\lambda + \frac{1}{2}\sqrt{\frac{2}{3}} q_{eff}. \quad (25)$$

The helicity amplitudes of (6) can be given in a simple factorized form as

$$A_M = F^S A_M^S(q_{eff}) + F^C A_M^C(q_{eff}), M = \frac{1}{2}, \frac{3}{2}, \quad (26)$$

where $A_M^S(q_{eff})$ and $A_M^C(q_{eff})$ are the non relativistic transition matrix elements between the nucleon and the resonance for the spin (S) and the transverse convective current (C) respectively. The non relativistic helicity amplitudes have the standard expressions

$$A_M^S = \frac{1}{2m}\sqrt{\frac{2\pi\alpha}{k_W}} \langle \phi_R | 3e(3) \big(\boldsymbol{\sigma}_3 \times \boldsymbol{q}\big)_+ e^{iqz_3^*} | \phi_N \rangle, \quad (27)$$

$$A_M^C = \frac{1}{2m}\sqrt{\frac{2\pi\alpha}{k_W}} \langle \phi_R | 3e(3) 2\boldsymbol{p}_{3+}^{*\perp} e^{iqz_3^*} | \phi_N \rangle, \quad (28)$$

where $|\phi_{N/R}\rangle$ represents the nucleon/resonance non relativistic wave function. We observe that, according to (26) and (15), the non relativistic matrix elements of (27) and (28) are to be calculated as functions of q_{eff}. The multiplicative factors in (26) are

$$F^S = \gamma^2 (t_S)^2 t_I g_S, \quad (29)$$

$$F^C = \gamma (t_S)^2 t_I h_C, \quad (30)$$

with γ as in (19) and

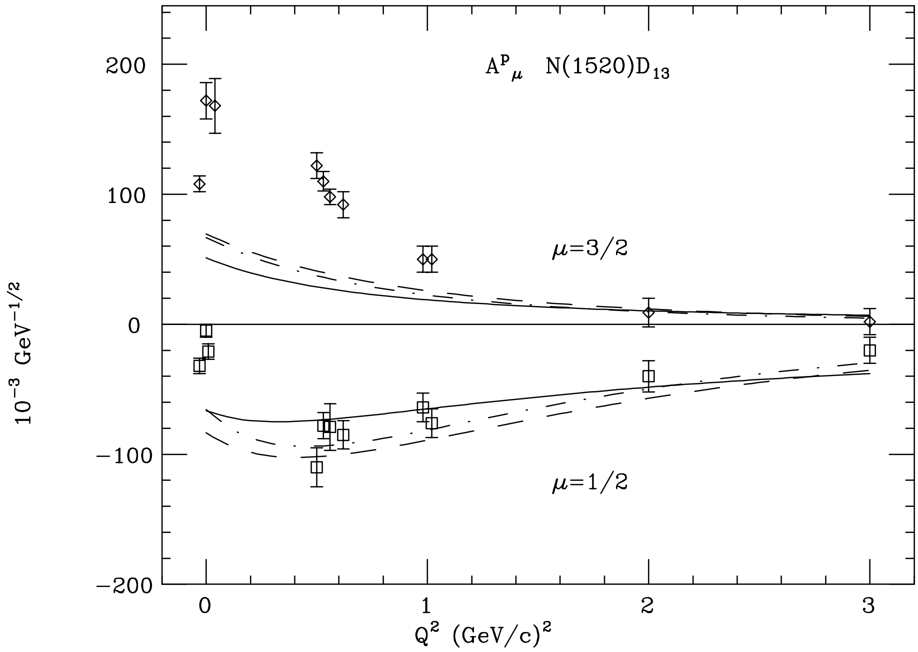

Fig. 1. Comparison between the experimental data for the helicity amplitudes $A^p_{3/2}$,$A^p_{1/2}$ for the $D_{13}(1520)$-resonance and the calculations with the relativistic corrections (full curve). The data are from the compilation of [31]. In the figure we report also the non relativistic calulations in the EVF (dashed curve) and in the Breit frame (dot-dashed curve)

$$t_S = \gamma\big[\eta_S - \frac{1}{6}\frac{vq_{eff}}{m}\big], \tag{31}$$

$$t_I = \frac{1}{\gamma}\frac{1}{\eta_I + \frac{1}{3}\frac{vq_{eff}}{m}}, \tag{32}$$

$$g_S = \frac{2}{3} + \frac{2m\eta_I}{M_R + M_N}, \tag{33}$$

$$h_C = \gamma\big[1 + \frac{1}{3}\frac{vq_{eff}}{m}\frac{1}{\eta_I + 1}\big], \tag{34}$$

$$\eta_S = \big[\frac{1}{36}\frac{q^2_{eff}}{m^2} + 1\big]^{\frac{1}{2}}, \tag{35}$$

$$\eta_I = \big[\frac{1}{9}\frac{q^2_{eff}}{m^2} + 1\big]^{\frac{1}{2}}, \tag{36}$$

with v as in (21). The coefficients t_S, t_I, η_S, η_I and g_S are the generalization for the inelastic transitions of the corresponding quantities introduced for the elastic case [16]. Within our approximations, we note that the relativistic corrections introduce two kind of modifications with respect to the non relativistic treatment: a multiplicative factor coming from the expansion of the quark spinors and the substitution of the momentum transfer $\boldsymbol{q}$ with the effective momentum $\boldsymbol{q}_{eff}$ in the non relativistic matrix elements. The latter replacement is in agreement with what was previously proposed by [30].

4 Results and comparison with experimental data

The matrix elements of (26), (27) and (28) can be calculated using as input the wave fuctions obtained in a non relativistic quark model. We present the results for the three-body force hypercentral potential [17] introduced in Sec. 2, which has been already used for the description of the spectrum [17,6], the photocouplings [29,6] and the elastic form factors with relativistic corrections [16]. We perform the calculations for the negative parity resonances, choosing those for which there are some experimental data available, namely the $D_{13}(1520)$, $S_{11}(1535)$, $S_{11}(1650)$, $S_{31}(1620)$ and $D_{33}(1700)$ states. The results are given in Figs. 1, 2, 3, 4, 5 and 6. We report the relativistic form factors of (26) in the EVF, compared with the results without relativistic correction in the same frame. For comparison we give also the non relativistic transition form factors in the Breit system [7].

We can observe from the various figures that the relativistic corrections modify slightly the high Q^2 behaviour, which remains in agreement with data. On the contrary, the relativistic corrections give a significant contribution at low Q^2, as already observed by [5]. It is interesting to observe that, even if one takes into account the relativistic kinematics, there still remains a strong discrepancy with the experimental data at low Q^2. This fact is in our opinion an indication that the present description of the e.m excitation of baryons has some deficiency. The problem is not that of finding a better 3-quark wave functions, as proved by the similar results obtained with different constituent quark models [4,9,12,14,7,6]. Actually some fundamental mechanism is lacking, as for instance the production of $q\overline{q}$ pairs and/or sea-quark effects.

In the figures the non relativistic calculations in the Breit frame are not drastically different from the non relativistic ones in the EVF.

For the electromagnetic excitations, one can calculate the transition radius. With the constituent quark model

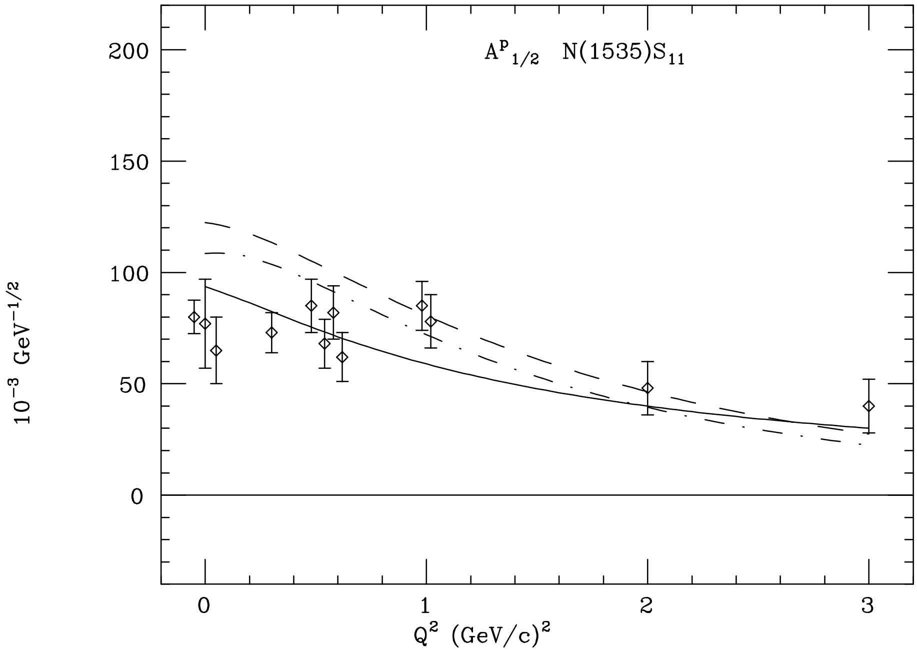

Fig. 2. Same as Fig. 1, but for the helicity amplitude $A^p_{1/2}$ of the $S_{11}(1535)$-resonance

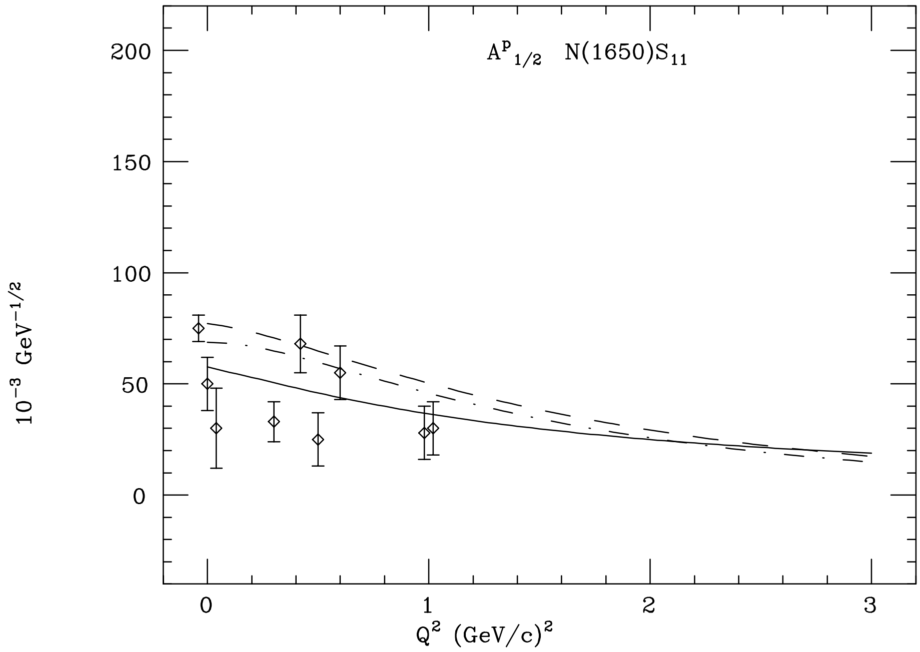

Fig. 3. Same as in Fig. 1, but for the helicity amplitude $A^p_{1/2}$ of the $S_{11}(1650)$-resonance

we are using, the radius for the transition to the negative parity resonances is about 17% higher than the elastic root mean square radius of the proton [6]. The relativistic corrections given in (26) produce a further increase which is of the order of 20% for both the spin and the convection form factors. Here again the relativistic corrections lead to larger radii as already observed for the elastic form factors [16].

5 Conclusions

We have calculated the relativistic corrections to the helicity amplitudes for the electromagnetic excitation of the negative parity nucleon resonances. We have used a simplified and preliminary approach which leads to simple analytical expressions. This method has been already applied to the calculation of the elastic charge and magnetic form factors of the nucleon [16]. We have used as

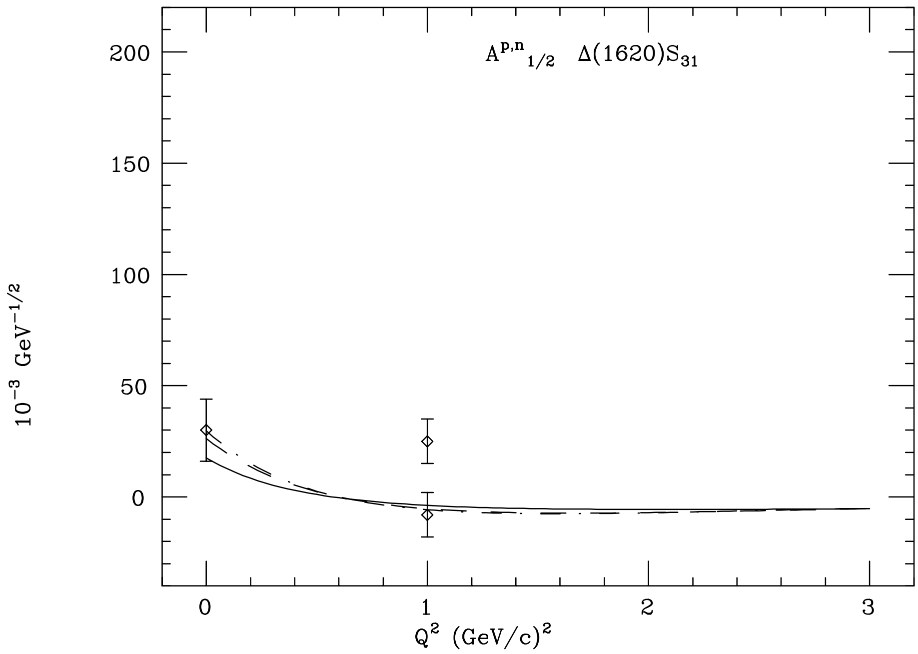

Fig. 4. Same as Fig. 1, but for the helicity amplitude $A^p_{1/2}$ of the $S_{31}(1620)$-resonance

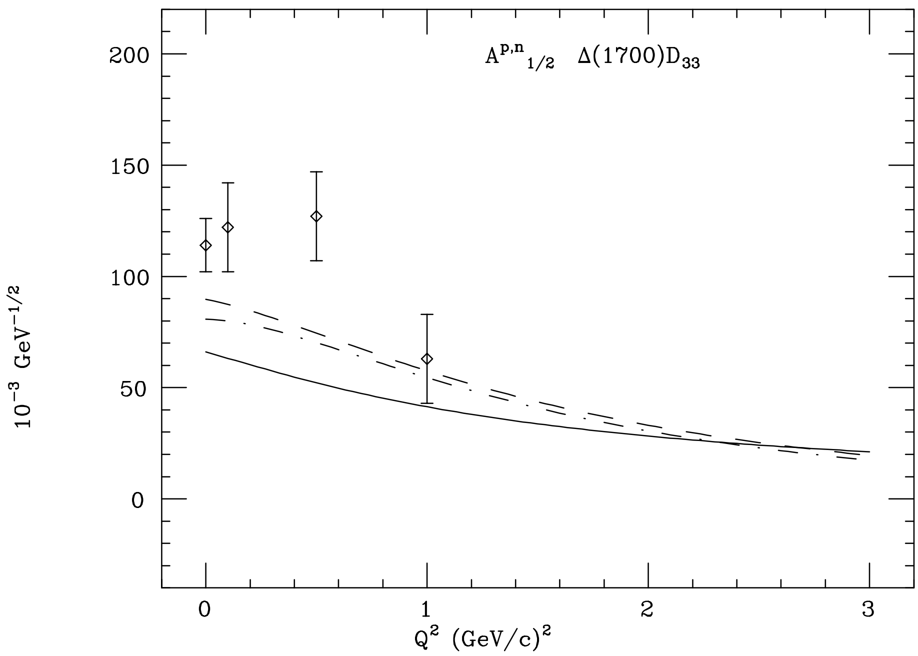

Fig. 5. Same as Fig. 1, but for the helicity amplitude $A^p_{1/2}$ of the $D_{33}(1700)$-resonance

input a non relativistic Constituent Quark Model with three-body forces in the hypercentral approach [17]. We have seen that at variance with what happens in the elastic case [16], the relativistic corrections to the transition form factors are not so important and in particular they are not sufficient to explain the discrepancy with data at low Q^2. It should be noted that we have here considered only the relativistic corrections coming from the Lorentz boosts and not the dynamical relativistic effects, such as for instance pair production, which are expected to be important at low Q^2.

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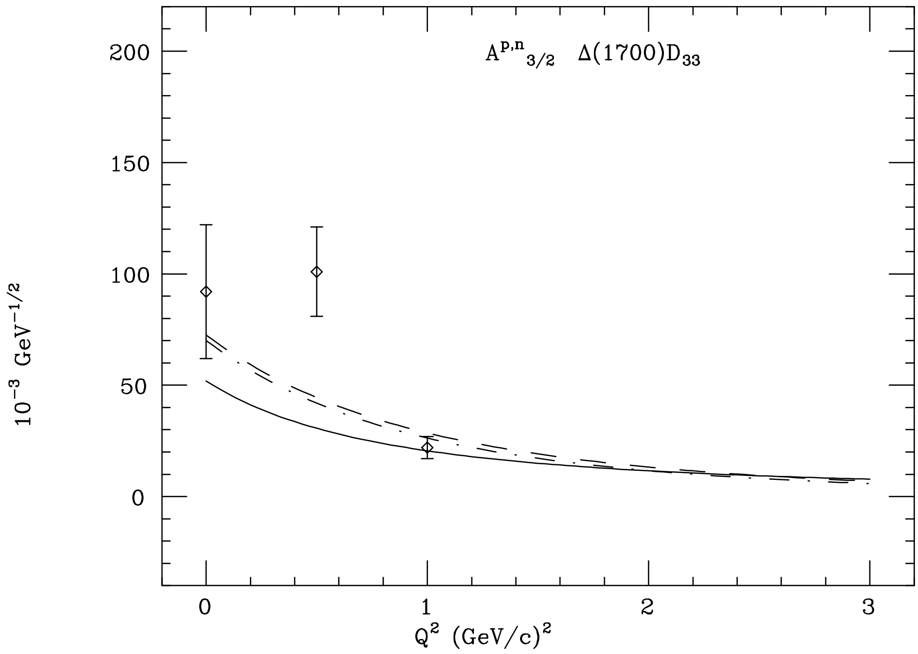

Fig. 6. Same as in Fig. 1, but for the helicity amplitude $A^p_{3/2}$ of the D_{33}(1700)-resonance

References

1. N. Isgur and G. Karl, Phys. Rev. **D18**, 4187 (1978); **D19**, 2653 (1979)
2. M.M. Giannini, Rep. Prog. Phys. **54**, 453 (1991) and references quoted therein
3. R. Bijker, F. Iachello and A. Leviatan, Ann. Phys. (N.Y.) **236**, 69 (1994)
4. R. Bijker, F. Iachello and A. Leviatan, Phys. Rev. **C54**, 1935 (1996)
5. S. Capstick and B.D. Keister, Phys. Rev. **D51**, 3598 (1995)
6. E. Santopinto, F. Iachello and M.M. Giannini, Nucl. Phys. **A623**, 100c (1997); Eur. Phys. J. **A1**, 307 (1998)
7. M. Aiello, M.M. Giannini and E. Santopinto, J. Phys. G: Nucl. Part. Phys. **24**, 1 (1998)
8. F. E. Close and Z. Li, Phys. Rev. **D42**, 2194 (1990); **D42**, 2207 (1990)
9. S. Capstick, Phys. Rev. **D46**, 2864 (1992)
10. A. Buchmann, E. Hernandez and K. Yazaki, Phys. Lett. **B 269**, 35 (1991); Nucl. Phys. **A 569**, 661 (1994)
11. E. Perazzi, M. Radici and S. Boffi, Nucl. Phys. **A 614**, 346 (1997)
12. F. Cardarelli, E. Pace, G. Salmè and S. Simula, Phys. Lett. **B 357**, 267 (1995)
13. A. Szczepaniak, Ji Chueng-Ryong and S.R.Cotanch, Phys. Rev. **C 52**, 2738 (1995)
14. M. Warns, H. Schröder, W. Pfeil and H. Rollnik, Z. Phys. **C 45**, 613 (1990) and 627 (1990); M. Warns, W. Pfeil and H. Rollnik, Phys. Rev. **C 42**, 2215 (1990)
15. M. De Sanctis and D. Prosperi, Il Nuovo Cimento **A 98**, 621 (1987)
16. M. De Sanctis, E. Santopinto and M.M. Giannini, Eur. Phys. J. **A1**, 187 (1998)
17. M. Ferraris, M.M. Giannini, M. Pizzo, E. Santopinto and L. Tiator, Phys. Lett. **B364**, 231 (1995)
18. R.A. Krajcik and L. L. Foldy, Phys. Rev. **D 12**, 1700 (1975)
19. J.L. Friar, Ann. Phys. (N.Y.)**81**, 332 (1973); Nuc. Phys. **A 264**, 455 (1976)
20. J. Ballot and M. Fabre de la Ripelle, Ann. of Phys. (N.Y.) **127**, 62 (1980)
21. M. Fabre de la Ripelle, in "Models and Methods in Few-Body Physics" (L.S. Ferreira, A. C. Fonseca and L. Streit eds.) Lecture Notes in Physics 273, Springer (Berlin), 1987, p. 283
22. M. Fabre de la Ripelle and J. Navarro, Ann. Phys. (N.Y.) **123**, 185 (1979)
23. J.-M. Richard, Phys. Rep. **C 212**, 1 (1992)
24. J. Leal Ferreira and P. Leal Ferreira, Lett. Nuovo Cimento vol. III, 43 (1970)
25. M.M. Giannini, Nuovo Cimento **A76**, 455 (1983); D. Drechsel, M.M. Giannini and L. Tiator, in "The Three-Body Force in the Three-Nucleon System", eds. B.L. Berman and B.F. Gibson, Lecture Notes in Physics bf260, 509 (1986); Few-Body Syst. Suppl. **2**, J.-L. Ballot and M. Fabre de la Ripelle eds., 448 (1987)
26. E. Santopinto, M.M. Giannini and F. Iachello, in "Symmetries in Science VII", ed. B. Gruber, Plenum Press, New York, 445 (1995); F. Iachello, in "Symmetries in Science VII", ed. B. Gruber, Plenum Press, New York, 213 (1995)
27. H.J. Lipkin, Rivista Nuovo Cimento I (volume speciale), 134 (1969)
28. H. Høgaasen and J.-M. Richard, Phys. Lett. **B124**, 520 (1983)
29. M. Aiello, M. Ferraris, M.M. Giannini, M. Pizzo and E. Santopinto, Phys. Lett. **B387**, 215 (1996)
30. F. Foster and G. Hughes, Z. Phys. **C14**, 123 (1982)
31. V. Burkert, private communication